

Anomalous Exponent of the Spin Correlation Function of a Quantum Hall Edges

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The charge and spin correlation functions of partially spin-polarized edge electrons of a quantum Hall bar are studied using effective Hamiltonian and bosonization techniques. In the presence of the Coulomb interaction between the edges with opposite chirality we find a different crossover behavior in spin and charge correlation functions. The crossover of the spin correlation function in the Coulomb dominated regime is characterized by an anomalous exponent, which originates from the finite value of the effective interaction for the spin degree of freedom in the long wavelength limit. The anomalous exponent may be determined by measuring nuclear spin relaxation rates in a narrow quantum Hall bar or in a quantum wire in strong magnetic fields.

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Edges of quantum Hall (QH) bars [1–4] have attracted a considerable attention recently. This is because they provide clean experimental verifications of many interesting theoretical predictions of Luttinger liquids [5,6]. Recent investigations have shown that the long-range Coulomb interaction brings new effects into these systems [7–10]. In *spin-polarized* systems it introduces a crossover from a Luttinger/Fermi liquid (power law) regime to a regime where the inter-edge Coulomb interaction dominates. A good example is the tunneling conductance at filling factor $\nu \leq 1$ between the quantum Hall edges of opposite chirality. It is described by [9]

$$G \sim \begin{cases} \left(\frac{T_0}{T}\right)^{2(1-1/\nu)}, & T > T_0 \\ \left(\frac{T_0}{T}\right)^2 \exp\left\{-\frac{2\sqrt{2}\chi}{3\nu} \left(\ln \frac{T_0}{T}\right)^{3/2}\right\}, & T < T_0, \end{cases} \quad (1)$$

where T_0 is the cross-over temperature scale. Similar cross-over exists in the CDW correlations [7,11,12]

$$C(x) \sim \begin{cases} (x/\ell)^{-2/\nu}, & x < W \\ \cos(2k_F x) \exp\left(-\frac{2}{\nu} \sqrt{\alpha \ln(x^2 \beta)}\right), & x > W, \end{cases} \quad (2)$$

where ℓ , W , and k_F are the magnetic length, the width of the Hall bar, and the Fermi wavevector.

In *partially* spin-polarized QH edges correlation functions are expected to exhibit interesting properties. Since the guiding center of the single particle wavefunction depends on the value of the wavevector, the wavefunctions of spin-up and -down electrons at the Fermi wavevectors are spatially separated. Moreover, spin and charge correlation functions are expected to behave differently since spin and charge separate [10]. We have investigated how spin and charge correlation functions behave as the transverse width of the Hall bar changes. We find that, although charge correlation functions are similar to the result given in Eq.(2), the spin correlation functions acquire *anomalous* exponents α_{2k_F} and α_0 . The imaginary part of transverse spin-spin correlation functions $S(\Omega)$ consist of inter- and intra-branch terms:

$$S_{\text{inter}}(\Omega) \propto \begin{cases} \Omega, & \Omega > \Omega_{\text{cr}} \\ \Omega^{\alpha_{2k_F}} e^{-\beta_{2k_F} [\ln \frac{v_\sigma}{|\Omega|d}]^{1/2}}, & \Omega < \Omega_{\text{cr}}, \end{cases} \quad (3)$$

$$S_{\text{intra}}(\Omega) \propto \begin{cases} \Omega, & \Omega > \Omega_{\text{cr}} \\ \Omega^{1+\alpha_0} e^{-\beta_0 [\ln \frac{v_\sigma}{|\Omega|d}]^{1/2}}, & \Omega < \Omega_{\text{cr}}. \end{cases} \quad (4)$$

Here $v_{\sigma(\rho)}$, d , $\beta_{0(2k_F)}$, and Ω_{cr} are some appropriate velocity, length, dimensionless constant, and crossover frequency. In the low energy limit the inter-branch contribution dominates over the intra-branch term since $\alpha_{2k_F} < 1 + \alpha_0$. However, the prefactor of the inter-branch term turns out to decay very rapidly as a function of the width of the Hall bar. Consequently, the intra-branch term is more relevant in wide Hall bars. The anomalous exponent α_0 goes to zero in the limit where the width of the Hall bar goes to infinity, and an expression similar to Eqs. (1) and (2) is recovered. For a narrow Hall bar or quantum wire in strong magnetic fields the anomalous exponent α_{2k_F} is significant since the edge separation is not negligible compared to the transverse width. The presence of the anomalous exponent in the transverse spin correlation function may be verified experimentally by measuring nuclear spin relaxation rates in a quantum wire in strong magnetic fields. In both expressions for the spin correlation function the power laws in Ω originate from the spin sector. The anomalous exponent emerges because the difference between spin-up and down electron wavefunctions at the respective Fermi wavevectors makes the effective interaction between the spin degrees of freedom of the *opposite* edges finite in the long wavelength limit. This effect is absent in zero magnetic field and is unique to partially spin-polarized edges.

We adopt the following model for a narrow Hall bar (or equivalently a quantum wire in strong magnetic fields [13]). When the transverse motion is confined by a parabolic potential the single-electron energy levels are

given by $E_{n,k} = (n+1/2)\Omega_0 + \frac{\hbar^2 k^2}{2m}$, where the enhanced longitudinal mass is $m = m^*(\Omega_0/\omega_0)^2$. The subband energy spacing is $\Omega_0 = \sqrt{\omega_c^2 + \omega_0^2}$, where $\omega_c = eB/m^*c$ is the cyclotron energy and ω_0 is the frequency of the harmonic potential. In this model, the degree of spin-splitting, i.e. the distance between the guiding centers of spin-up and -down electrons at the Fermi wavevectors [14], can be easily tuned by the magnetic field since the effective longitudinal mass of the electron depends on the value of the magnetic field [15]. The electronic wavefunction at the Fermi wavevectors $k_{F,r,s}$ of the lowest magnetic subband is given by

$$\phi_{r,s}(x, y) = \frac{e^{ik_{F,r,s}x}}{\pi^{1/4} \tilde{\ell}^{1/2}} \exp\left(-\frac{(y - R^2 k_{F,r,s})^2}{2\tilde{\ell}^2}\right), \quad (5)$$

where $\tilde{\ell} = (\hbar/m\Omega_0)^{1/2}$, $R^2 = \hbar\omega_c/(m\Omega_0^2)$, $r = R(L)$ for the right (left) branch, and y is the transverse coordinate [16]. Because of the large longitudinal mass at fields where $\omega_c > \omega_0$ all the electrons can be accommodated in the lowest magnetic subband. Each branch r consists of spin-up and -down edges ($s = \uparrow, \downarrow$). The intra branch electron-electron interactions are given by

$$V_{q,\parallel}^{\text{intra}} = -\frac{2e^2}{\epsilon} \ln \frac{\gamma q a_\parallel}{2}, \quad V_{q,\perp}^{\text{intra}} = -\frac{2e^2}{\epsilon} \ln \frac{\gamma q a_\perp}{2} \quad (6)$$

and the inter branch electron-electron interactions are given by

$$V_{q,\parallel,s}^{\text{inter}} = \frac{2e^2}{\epsilon} K_0(qW_{\parallel,s}), \quad V_{q,\perp}^{\text{inter}} = \frac{2e^2}{\epsilon} K_0(qW_\perp). \quad (7)$$

Here $W_{\parallel,s} = |k_{F,R,s} - k_{F,L,s}|R^2$ and $W_\perp = |k_{F,R,s} - k_{F,L,-s}|R^2$, and $K_0(x)$ is the modified Bessel function, which behaves like $-\ln x$ in the limit of small x . The constants a_\parallel and a_\perp are the length scales comparable to the width of the quantum wire, and $v_{F,s}$ are the Fermi velocities [17].

It is convenient to use charge and spin density operators $\rho_{r,q} = \frac{1}{\sqrt{2}}(\rho_{r,q,\uparrow} + \rho_{r,q,\downarrow})$ and $\sigma_{r,q} = \frac{1}{\sqrt{2}}(\rho_{r,q,\uparrow} - \rho_{r,q,\downarrow})$. The bosonized Hamiltonian can be written as a sum of the charge, spin, and mixed terms $H = H_\rho + H_\sigma + \delta H$, where

$$H_\rho = \frac{\pi v_F}{L} \sum_{r,q \neq 0} : \rho_{r,q} \rho_{r,-q} : + \frac{1}{L} \sum_{r,q} V_{\rho,q}^{\text{intra}} : \rho_{r,q} \rho_{r,-q} : + \frac{2}{L} \sum_q V_{\rho,q}^{\text{inter}} \rho_{R,q} \rho_{L,-q}, \quad (8)$$

$$H_\sigma = \frac{\pi v_F}{L} \sum_{r,q \neq 0} : \sigma_{r,q} \sigma_{r,-q} : + \frac{1}{L} \sum_{r,q} V_{\sigma,q}^{\text{intra}} : \sigma_{r,q} \sigma_{r,-q} : + \frac{2}{L} \sum_q V_{\sigma,q}^{\text{inter}} \sigma_{R,q} \sigma_{L,-q}, \quad (9)$$

and

$$\delta H = 2\delta v_F \frac{\pi}{L} \sum_{r,q} \sigma_{r,q} \rho_{r,-q} + \frac{1}{L} \sum_q \delta V_q^{\text{inter}} [\sigma_{R,q} \rho_{L,-q} + \sigma_{L,q} \rho_{R,-q}], \quad (10)$$

with

$$\begin{aligned} V_{\rho(\sigma),q}^{\text{intra}} &= \frac{1}{2} (V_{q,\parallel}^{\text{intra}} \pm V_{q,\perp}^{\text{intra}}), \\ V_{q,\parallel}^{\text{inter}} &= \frac{1}{2} (V_{q,\parallel,\uparrow}^{\text{inter}} + V_{q,\parallel,\downarrow}^{\text{inter}}), \\ \delta V_q^{\text{inter}} &= \frac{1}{2} (V_{q,\parallel,\uparrow}^{\text{inter}} - V_{q,\parallel,\downarrow}^{\text{inter}}), \\ V_{\rho(\sigma),q}^{\text{inter}} &= \frac{1}{2} (V_{q,\parallel}^{\text{inter}} \pm V_{q,\perp}^{\text{inter}}), \\ v_F &= \frac{1}{2} (v_{F\uparrow} + v_{F\downarrow}), \quad \delta v_F = \frac{1}{2} (v_{F\uparrow} - v_{F\downarrow}). \end{aligned} \quad (11)$$

Note that in the effective Hamiltonian, the charge and spin degrees of freedom are separated except in δH [10].

In the computation of correlation functions the *phase fields* ($\theta_{\rho,\sigma}$, $\phi_{\rho,\sigma}$) formulation is more convenient. In momentum space, they are defined as

$$\begin{aligned} -\frac{iq}{\pi} \phi_\rho &= \rho_R + \rho_L, \quad -\frac{iq}{\pi} \phi_\sigma = \sigma_R + \sigma_L, \\ \frac{iq}{\pi} \theta_\rho &= \rho_R - \rho_L, \quad \frac{iq}{\pi} \theta_\sigma = \sigma_R - \sigma_L. \end{aligned} \quad (12)$$

We can derive the effective action corresponding to the Hamiltonian by the standard procedure. The action in imaginary time and at zero temperature is

$$\begin{aligned} S = \frac{1}{2\pi} \int \frac{d\omega dk}{(2\pi)^2} &\left[(\omega^2 \frac{v_{\sigma-}}{v_{\rho-} - v_{\sigma-} - g_-^2} + v_{\rho+} k^2) \phi_\rho \phi_\rho \right. \\ &+ (\omega^2 \frac{v_{\rho-}}{v_{\rho-} - v_{\sigma-} - g_-^2} + v_{\sigma+} k^2) \phi_\sigma \phi_\sigma \\ &\left. + 2(-\omega^2 \frac{g_-}{v_{\rho-} - v_{\sigma-} - g_-^2} + g_+ k^2) \phi_\sigma \phi_\rho \right], \end{aligned} \quad (13)$$

where

$$\begin{aligned} v_{\rho\pm} &= v_F + \frac{V_{\rho,q}^{\text{intra}}}{\pi} \pm \frac{V_{\rho,q}^{\text{inter}}}{\pi}, \\ v_{\sigma\pm} &= v_F + \frac{V_{\sigma,q}^{\text{intra}}}{\pi} \pm \frac{V_{\sigma,q}^{\text{inter}}}{\pi}, \\ g_\pm &= \delta v_F \pm \frac{\delta V_q^{\text{inter}}}{\pi}. \end{aligned} \quad (14)$$

For later convenience, we define $W_\parallel = (W_{\parallel\uparrow} W_{\parallel\downarrow})^{1/2}$ and $v_0 = \frac{2e^2}{\hbar\pi\epsilon}$. The Eq.(13) can be also expressed in terms of conjugate phase fields by simply replacing $v_{\rho\pm} \rightarrow v_{\rho\mp}$, $v_{\sigma\pm} \rightarrow v_{\sigma\mp}$, and $g_\pm \rightarrow g_\mp$. We will use the action (13) and its conjugate action for the computation of correlation functions. The propagators of phase fields

can be obtained by inverting kernel matrices of (13) and its conjugate action. Finally, we need the explicit expression of electron operators in terms of phase fields [12]

$$\psi_{r,s}(x,y) = \phi_{r,s}(x,y) e^{-\frac{i}{\sqrt{2}}(r(\phi_\rho+s\phi_\sigma)+(\theta_\rho+s\theta_\sigma))}. \quad (15)$$

It is slightly different from the bosonization formula of a truly 1D system because the left and right edges are spatially separated by the width of the quantum wire.

Let us consider the correlation function of the transverse spin operator. The transverse spin operator is

$$\begin{aligned} \hat{S}^+(x) &= \int dy \sum_{r,r'=R,L} \psi_{r\uparrow}^\dagger(x,y) \psi_{r'\downarrow}(x,y) \\ &= C_0(x) e^{-i\sqrt{2}\theta_\sigma(x)} \cos \sqrt{2}\phi_\sigma(x) \\ &\quad + C_{2k_F}(x) e^{-i\sqrt{2}\theta_\sigma(x)} \cos \sqrt{2}\phi_\rho(x), \end{aligned} \quad (16)$$

where $C_{0(2k_F)}(x) = e^{-i(k_F\uparrow \mp k_F\downarrow)x} e^{-\frac{(k_F\uparrow \mp k_F\downarrow)^2 R^4}{4\ell^2}}$. The first term of Eq.(16) is the intra-branch contribution, and the second term is the inter-branch contribution ($2k_F$ component). Note that the intra-branch spin operator is composed entirely of spin bosons ($\phi_\sigma(x)$ and $\theta_\sigma(x)$) while the inter-branch spin operator is composed of both the charge $\phi_\rho(x)$ and spin $\theta_\sigma(x)$ degrees of freedom. When the off-diagonal elements of the action are negligible the correlation function of the intra-branch spin operator will only reflect the spin degree of freedom.

The inter- and intra-branch terms of the imaginary part of the transverse correlation function are given in Eqs. (3) and (4) [18]. The the anomalous exponent α_{2k_F} is given by [19]

$$\alpha_{2k_F} = \left[\frac{\frac{v_F}{v_0} + \frac{1}{2} \ln \frac{a_\perp W_\perp}{a_\parallel W_\parallel}}{\frac{v_F}{v_0} + \frac{1}{2} \ln \frac{a_\perp W_\parallel}{a_\parallel W_\perp}} \right]^{1/2} - 1. \quad (17)$$

The other anomalous exponent α_0 is

$$\alpha_0 = \left[\frac{\frac{v_F}{v_0} + \frac{1}{2} \ln \frac{a_\perp W_\perp}{a_\parallel W_\parallel}}{\frac{v_F}{v_0} + \frac{1}{2} \ln \frac{a_\perp W_\parallel}{a_\parallel W_\perp}} \right]^{1/2} + [W_\parallel \leftrightarrow W_\perp]^{1/2} - 2. \quad (18)$$

The crossover frequency Ω_{cr} is roughly $v_\rho/\sqrt{W_\parallel W_\perp}$ and the anomalous exponents are always non-negative since $\frac{W_\perp}{W_\parallel} > 1$. The amplitude of the $2k_F$ component of correlation function $|C_{2k_F}(x)|^2$ is explicitly given by $e^{-2R^4 k_F^2 / \ell^2} = \exp \left[-4 \frac{E_F}{\omega_0} \frac{\omega_0 \omega_c^2}{\Omega_0^3} \right]$. For ordinary QH bars this amplitude is negligibly small [12], but for a narrow QH bar or a quantum wire in strong magnetic fields it is of order one for reasonable values of E_F, ω_0 , and B . For reasons given in the second paragraph α_{2k_F} is experimentally more relevant than α_0 . From Eq. (17) we see that when $V_{\sigma,q}^{\text{inter}} = 0$ ($W_\parallel = W_\perp$) the anomalous exponent α_{2k_F} vanishes irrespective of a_\parallel and a_\perp . This

means only the $V_{\sigma,q}^{\text{inter}}$ interaction in the spin part of the Hamiltonian H_σ can give rise to an anomalous exponent. In the long wavelength limit this interaction takes a *finite* value since it is given by the *difference* between two modified Bessel functions. In contrast the effective interactions $V_{\rho,q}^{\text{intra}}$ and $V_{\rho,q}^{\text{inter}}$ of H_ρ diverge in the same limit. The numerical value of α_{2k_F} is dependent on the detailed shape of the confining potential, and therefore, is not universal.

The correlation function of the longitudinal spin operator is very similar to that of transverse one. The same crossover exists, and the inter-branch contribution dominates over the intra-branch contribution in the Coulomb regime. The anomalous exponent α_{2k_F} is obtained by replacing W_\perp with W_\parallel in Eq. (17) [20]. Because $W_\perp > W_\parallel$ the exponent α_{2k_F} is now negative, which signals the SDW-like ground state at zero temperature. Recall that in our system the spin SU(2) symmetry is broken by magnetic field, and it is natural that the transverse and longitudinal spin correlation functions have different anomalous exponents. We have also calculated the cross-over behavior of charge correlation functions, and find that it is almost identical with those of quantum wires at zero magnetic field and spin-polarized edges at filling factor 1 [7,11]. In quantum wires one can observe upon bosonization that the long-range Coulomb interaction couples only to the charge degree of freedom [11]. In this case an anomalous exponent is absent in the spin sector.

We now discuss the experimental relevance of the frequency dependence of the transverse spin correlation function. We believe that NMR measurements of a quantum wire in strong magnetic fields should demonstrate the presence of the anomalous exponent since the transverse spin correlation function is directly related to the nuclear spin relaxation rate. (NMR measurements of quantum Hall edges have been carried out recently [21]). The effect of the spin-splitting on the anomalous exponent would be most strong when the separation between spin-up and -down Fermi edges is large. In such a system the widths W_\parallel and W_\perp would be rather different and should yield a significant value of the anomalous exponent. To get an estimate of α_{2k_F} , we expand Eq.(17), and find $\alpha_{2k_F} \approx \frac{4v_0}{v_F} \ln \frac{W_\perp}{W_\parallel}$. Since v_F is of order v_0 in the absence of a magnetic field it can be made significantly smaller than v_0 by applying a strong magnetic field [17]. From this we estimate that $\alpha_{2k_F} \sim 0.1$ is reasonable.

In conclusion, we have shown that the charge and spin correlation functions of spin-polarized edge states behave qualitatively differently. This effect is unique to 1D systems in strong magnetic fields and is based on the novel property of shifting of the guiding center of the Landau level wavefunction with the change in the single-particle quantum number. In the long wavelength limit the effective interaction for the spin degree of freedom takes a

finite value while the effective interaction for the charge degree of freedom is infinitely strong. As a consequence, an anomalous exponent appears in the spin sector. The presence of the anomalous exponent may be tested experimentally in a narrow QH bar or in a quantum wire in strong magnetic fields.

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